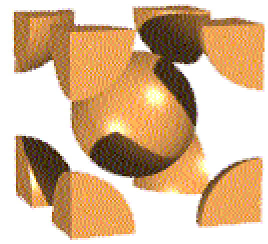
Solving the Body-Centered Cubic Lattice Problem

Your textbook does an excellent job of walking you through the solution of a density problem in which the metal exhibits a face-centered cubic lattice. The same strategy can be applied to solving problems with the body-centered cubic lattice.

First, lets get a good look at a body-centered cubic lattice:

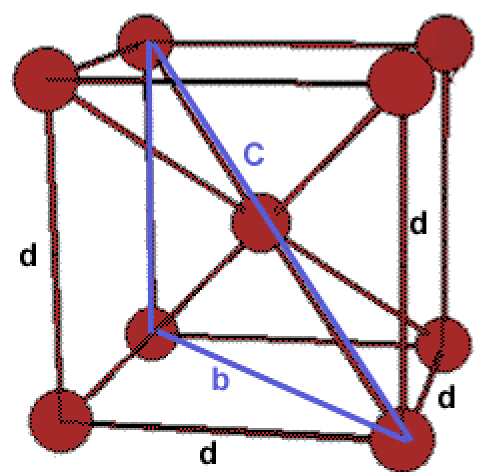


1. Notice that there is a central particle, surrounded by eight closest neighbors.
2. Each unit cell contains the particle at the center, plus the fractions of a particle at each corner. Since each of these corners represents 1/8 of a particle, the unit cell contains:

   $$1 + 8(1/8) = 2 \text{ particles}$$

   This is different than the face-centered cubic lattice that contains 4 particles per unit cell.

In order to calculate a density, we need to know the length of a side, d, of the cubic lattice, assuming that we have been given the particle radius, r. This is slightly trickier than the face-centered lattice, because our diagonal doesn't lie on the face of the cube, but instead it lies within the body of the cube. We will also assume that the particles come in contact with each other, unlike the drawing.



1. Lets first analyze the triangle in blue. The hypotenuse "c" contains one particle (2r) plus a single radius from each of two particles at the corners. Thus, the length of "c" assuming that the particles actually touch, is 4r.
2. Solving for the triangle in blue:

   $$c^2 = b^2 + d^2$$

3. Substituting,

   $$(4r)^2 = b^2 + d^2$$

4. We don't have a value for b, so we need to recognize it is also the hypotenuse of a right triangle:

   $$b^2 = d^2 + d^2$$

5. Substituting again,

   $$(4r)^2 = d^2 + d^2 + d^2$$

6. Simplifying,

   $$16r^2 = 3d^2$$

7. Solving for d,

   $$d = r\sqrt{\frac{16}{3}}$$

From this point, you may continue to solve the problem using the solution for the face-centered cubic lattice on pages 457 and 458. Remember when doing the density calculation that there are two atoms per unit cell in a body-centered cubic lattice as opposed to 4 atoms per unit cell in a face-centered cubic lattice.